Broadening of a nonequilibrium phase transition by extended structural defects

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We study the effects of quenched extended impurities on nonequilibrium phase transitions in the directed percolation universality class. We show that these impurities have a dramatic effect: they completely destroy the sharp phase transition by smearing. This is caused by rare strongly coupled spatial regions which can undergo the phase transition independently from the bulk system. We use extremal statistics to determine the stationary state as well as the dynamics in the tail of the smeared transition, and we illustrate the results by computer simulations.

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In nature, thermal equilibrium is more of an exception than the rule. In recent years, phase transitions between different nonequilibrium states have become a topic of great interest. A prominent class of nonequilibrium phase transitions separates active, fluctuating states from inactive, absorbing states where fluctuations cease entirely. These absorbing state transitions have applications ranging from physics to chemistry and to biology [1–4]. The generic universality class for absorbing state transitions is directed percolation (DP) [5]. According to a conjecture by Janssen and by Grassberger [6], all absorbing state transitions with a scalar order parameter, short-range interactions, and no extra symmetries or conservation laws belong to this class. Examples include the transitions in the contact process [7], catalytic reactions [8], interface growth [9], and turbulence [10]. However, despite its ubiquity in theory and simulations, clearcut experimental realizations of the DP universality class are strangely lacking [11]. The only verification so far seems to be found in the spatiotemporal intermittency in ferrofluidic spikes [12].

A possible reason for this apparent discrepancy is the impurities, i.e., quenched spatial disorder. According to the Harris criterion [13,14], the DP universality class is unstable against disorder, because the (spatial) correlation length exponent $\nu_\perp$ violates the inequality $\nu_\perp > 2/d$ for all spatial dimensionalities $d<4$. Indeed, in the corresponding field theory, spatial disorder leads to runaway flow of the renormalization group (RG) equations [15], destroying the DP behavior. Several other studies [16–19] agreed on the instability of DP against spatial disorder, but a consistent picture has been slow to evolve. Recently, Hooyberghs *et al.* applied the Hamiltonian formalism [20] to the contact process with spatial disorder [21]. Using the Ma-Dasgupta-Hu strong-disorder RG [22] these authors showed that the transition (at least for sufficiently strong disorder) is controlled by an exotic infinite-randomness fixed point with activated rather than the usual power-law scaling. In many real systems, the disorder does not consist of point defects but of dislocations, disordered layers, or grain boundaries. The effects of such extended defects are generically stronger than that of uncorrelated disorder, as has been shown by detailed studies of equilibrium systems ranging from the exactly solved McCoy-Wu model [23] and several RG studies [24–27] to the discovery of infinite-randomness critical behavior in the corresponding quantum Ising model [28].

In this paper, we show that extended defects have an even more dramatic effect on nonequilibrium phase transitions in the DP universality class; they destroy the sharp transition by smearing. This is caused by phenomena similar to but stronger than the usual Griffiths effects [14,29]: rare strongly coupled spatial regions can undergo the transition independently of the bulk system. In the tail of the smeared transition, the spatial density distribution is very inhomogeneous, with the average stationary density and the survival probability depending exponentially on the control parameter. The approach of the average density to this exponentially small stationary value occurs in two stages, a stretched exponential decay at intermediate times, followed by power-law behavior at late times. In the following, we derive these results for a disordered contact process, illustrate them by computer simulations, and discuss their generality and importance.

Our starting point is the clean contact process [7], a prototypical system in the DP universality class. It is defined on a d-dimensional hypercubic lattice. Each site $\mathbf{r}$ can be vacant or active, i.e. occupied by a particle. During the time evolution, particles are created at vacant sites at a rate $\lambda n/(2d)$ where n is the number of active nearest neighbor sites and the creation rate λ is the control parameter. Particles are annihilated at unit rate. For small λ, annihilation dominates, and the absorbing state without any particles is the only steady state. For large λ there is a steady state with finite particle density (active phase). The two phases are separated by a nonequilibrium phase transition in the DP universality class at $\lambda=\lambda_c^0$.

We introduce quenched spatial disorder by making the creation rate λ a random function of the lattice site. Extended impurities can be described by disorder perfectly correlated in d_c dimensions, but uncorrelated in the remaining $d_r=d-d_c$ dimensions. λ is thus a function of $\mathbf{r}_r$, which is the projection of the position vector $\mathbf{r}$ on the uncorrelated directions. For definiteness, we assume that the $\lambda(\mathbf{r}_r)$ have a binary probability distribution

$$P[\lambda(\mathbf{r}_r)] = (1-p)\,\delta(\lambda(\mathbf{r}_r)-\lambda) + p\,\delta(\lambda(\mathbf{r}_r)-c\lambda), \qquad (1)$$

where p and c are constants between 0 and 1. In other words, there are extended impurities of density p where the creation rate λ is reduced by a factor c.

Let us now consider the effects of rare disorder fluctuations in this system. In analogy to the Griffiths phenomenon [14,29], there is a small but finite probability for finding large spatial regions devoid of impurities. These rare regions can be locally in the active phase, even if the bulk system is still in the inactive phase. For the largest rare regions this starts to happen when λ crosses the clean critical point λ_c^0. Since the impurities in our system are extended, each rare region is infinite in d_c dimensions but finite in the remaining d_r dimensions. This is a crucial difference from systems with uncorrelated disorder, where the rare regions are finite. In our system, each rare region can therefore undergo a real phase transition *independently* of the rest of the system. Thus, those rare regions that are locally in the ordered phase will have a true nonzero stationary density, even if the bulk system is still in the inactive phase.

The resulting global phase transition is very different from a conventional continuous phase transition, where a nonzero order parameter develops as a collective effect of the entire system, accompanied by a diverging correlation length in all directions. In contrast, in our system, the order parameter develops very inhomogeneously in space with different parts of the system (i.e., different $\mathbf{r}_r$ regions) ordering independently at different λ. Correspondingly, the correlation length in the uncorrelated directions remains finite across the transition. This defines a smeared transition. Thus, extended impurities lead to a smearing of the DP phase transition.

We now use extremal statistics to derive the properties in the tail of the smeared transition, i.e., in the parameter region where a few active rare regions exist, but their density is so small that they can be treated as independent. We start with the stationary behavior. The probability w for finding a rare region of linear size L_r devoid of impurities is, up to preexponential factors, given by

$$w \sim \exp(-\tilde{p} L_r^{d_r}) \tag{2}$$

with $\tilde{p}=-\ln(1-p)$. As discussed above, such a region undergoes a true phase transition to the active phase at some $\lambda_c(L_r) > \lambda_c^0$. According to finite-size scaling [30],

$$\lambda_c(L_r) - \lambda_c^0 = A L_r^{-\phi}, \tag{3}$$

where ϕ is the clean (d-dimensional) finite-size scaling shift exponent and A is the amplitude for the crossover from a d-dimensional bulk system to a "slab" infinite in d_c dimensions but finite in d_r dimensions. If the total dimensionality $d=d_c+d_r<4$, hyperscaling is valid and $\phi=1/\nu_\perp$ which we assume from now on. Combining Eqs. (2) and (3) we obtain the probability for finding a rare region which becomes active at λ_c as

$$w(\lambda_c) \sim \exp[-B(\lambda_c - \lambda_c^0)^{-d_r \nu_\perp}] \tag{4}$$

for $\lambda_c - \lambda_c^0 \to 0+$. Here $B=\tilde{p} A^{d_r \nu_\perp}$. The total (average) density ρ at a certain λ is obtained by summing over all active rare regions, i.e., all regions with $\lambda_c < \lambda$. Since the functional dependence on λ of the density on any given active island is of power-law type it does not enter the leading exponentials but only the preexponential factors. Thus, the stationary density develops an exponential tail

$$\rho(\lambda) \sim \exp[-B(\lambda - \lambda_c^0)^{-d_r \nu_\perp}] \tag{5}$$

reaching the clean critical point λ_c^0. Analogous arguments can be made for the survival probability $P(\lambda)$ of a single seed site. If the seed site is on an active rare region it will survive with a probability that depends on λ with a power law. If it is not on an active rare region, the seed will die. To exponential accuracy the survival probability is thus also given by Eq. (5).

The local (spatial) density distribution in the tail of the smeared transition is very inhomogeneous. On active rare regions, the density is of the same order of magnitude as in the clean system. Away from these regions it decays exponentially. The typical local density ρ_{typ} can be estimated from the typical distance of any point from the nearest active rare region. From Eq. (4) we obtain

$$r_{\text{typ}} \sim \exp[B(\lambda - \lambda_c^0)^{-d_r \nu_\perp}/d_r]. \tag{6}$$

At this distance, the local density has decayed to

$$\rho_{\text{typ}} \sim e^{-r_{\text{typ}}/\xi_0} \sim \exp\{-C \exp[B(\lambda - \lambda_c^0)^{-d_r \nu_\perp}/d_r]\} \tag{7}$$

where ξ_0 is the bulk correlation length (which is finite and changes slowly across the smeared transition) and C is a constant. A comparison with Eq. (5) shows that the relation between the typical and the average density is exponential, $|\log \rho_{\text{typ}}| \sim \rho^{-1/d_r}$, indicating an extremely broad local density distribution.

We now turn to the dynamics in the tail of the smeared transition. The long-time decay of the density is dominated by the rare regions while the bulk contribution decays exponentially. According to finite-size scaling [30], the behavior of the correlation time ξ_t of a single rare region of size L_r in the vicinity of the clean bulk critical point can be modeled by

$$\xi_t(\Delta, L_r) \sim L_r^{(z\nu_\perp - \tilde{z}\tilde{\nu}_\perp)/\nu_\perp} \left|\Delta - A L_r^{-1/\nu_\perp}\right|^{-\tilde{z}\tilde{\nu}_\perp}. \tag{8}$$

Here $\Delta=\lambda-\lambda_c^0>0$, z is the d-dimensional bulk dynamical critical exponent, and $\tilde{\nu}_\perp$ and $\tilde{z}$ are the correlation length and dynamical exponent of a d_r-dimensional system. Let us first consider the time evolution of the density at $\lambda=\lambda_c^0$. For $\Delta=0$, the correlation time (8) simplifies to $\xi_t \sim L_r^z$. To exponential accuracy, the time dependence of the average density is obtained from

$$\rho(t) \sim \int dL_r \exp(-\tilde{p} L_r^{d_r} - Dt/L_r^z) \tag{9}$$

where D is a constant. Using the saddle point method to evaluate this integral, we find the leading long-time decay of the density to be given by a stretched exponential,

$$\ln \rho(t) \sim -t^{d_r/(d_r+z)}. \tag{10}$$

For $\lambda > \lambda_c^0$, we repeat the saddle point analysis with the full expression (8) for the correlation length. For intermediate times $t < t_x \sim (\lambda - \lambda_c^0)^{-(d_r+z)\nu_\perp}$, the decay of the average density is still given by the stretched exponential (10). For times larger than the crossover time t_x the system realizes that some of the rare regions are in the active phase and

contribute to a finite steady state density. The approach of the average density to this steady state value is characterized by a power law:

$$\rho(t) - \rho(\infty) \sim t^{-\psi}. \tag{11}$$

The value of ψ cannot be found by our methods since it depends on the neglected preexponential factors.

We now illustrate the smearing of the phase transition by the results of a computer simulation of a two-dimensional (2D) contact process with linear defects ($d_c = d_r = 1$). To reach the rather large system sizes necessary to observe exponentially rare events, we consider a version of the contact process with infinite-range couplings in the correlated direction (parallel to the impurities) but nearest-neighbor couplings in the uncorrelated direction (perpendicular to the defect lines). While this infinite-range model will not be quantitatively comparable to a short-range contact process, it provides a simple example for the smearing mechanism. Moreover, since the smearing relies only on a single rare region undergoing a true phase transition, we expect that the results will qualitatively valid for a short-range contact process too (with the appropriate changes to the exponents).

Because the couplings in the correlated direction are of infinite range, this dimension can be treated exactly in mean-field theory. This leads to a set of coupled local mean-field equations for the local densities ρ_x,

$$\frac{\partial}{\partial t}\rho_x = -\rho_x + \frac{\lambda(x)}{4}(1-\rho_x)(\rho_{x-1} + 2\rho_x + \rho_{x+1}). \tag{12}$$

These equations can easily be solved numerically. We study systems with several dilutions $p = 0.2, \ldots, 0.6$ and sizes of up to $L = 10^6$ in the uncorrelated direction; the impurity strength is $c = 0.2$ for all calculations.

To determine the stationary state we solve the equations $(\partial/\partial t)\rho_x = 0$ in a self-consistency cycle. Our results are summarized in Fig. 1. The top panel shows the total (average) density ρ for a clean ($p = 0$) and a dirty ($p = 0.3$) system. The clean system has the expected sharp phase transition at $\lambda = \lambda_c^0 = 1$ with the mean-field critical exponent $\beta = 1$. The data of the disordered system seem to suggest a transition at $\lambda \approx 1.04$. However, a closer inspection shows that the singularity is smeared. Note that the density data are essentially size independent. Therefore, the observed rounding cannot be due to finite-size effects. We conclude that the smearing is an intrinsic effect of the infinite system. For comparison with the analytical prediction (5), the center panel shows the logarithm of the total density as a function of $(\lambda - \lambda_c^0)^{-1/2}$ for different impurity concentrations p. Note that in our infinite-range model $\nu_\perp = 1/\phi = 1/2$. The data follow Eq. (5) over several orders of magnitude in ρ. Fits of the data to Eq. (5) are used to determine the decay constants B. The bottom panel of Fig. 1 shows that these decay constants depend linearly on $\tilde{p} = -\ln(1-p)$, as predicted.

To study the time evolution we numerically integrate the local mean-field equations (12), starting from a homogeneous initial state with $\rho = 1$. Figure 2 summarizes our results for a system of size $L = 10^6$ with dilution $p = 0.5$. The main panel shows a log-log plot of $\ln \rho$ vs t. This allows us to test

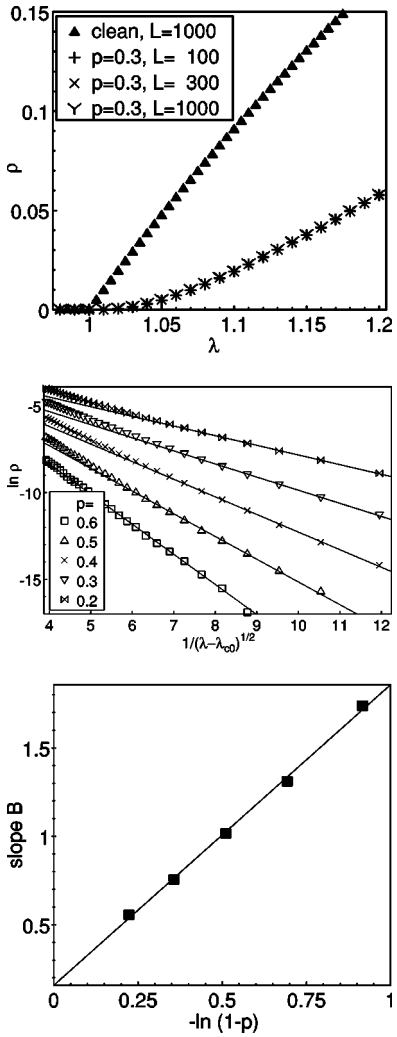

FIG. 1. Top: Overview of the steady state density of a clean ($p = 0$) and a diluted ($p = 0.3$) system. Center: Logarithm of the density as a function of $(\lambda - \lambda_c^0)^{-1/2}$ for several dilutions p and $L = 10^4$. The data are averages over 100 disorder realizations. The solid lines are fits to Eq. (5) with $d_r \nu_\perp = 1/2$. Bottom: Decay constant B as a function of $-\ln(1-p)$.

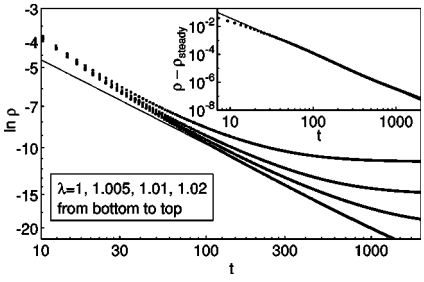

FIG. 2. Density ρ vs. time t for a system of size $L = 10^6$, dilution $p = 0.5$ and several λ (averages over 25 disorder realizations). Solid line: Fit of the $\lambda = 1$ data ($t > 100$) to Eq. (10) giving an exponent of approx. 0.32. Inset: Approach to the steady state density for $\lambda = 1.01$. Solid line: Fit of the data for $t > 100$ to (11), giving an exponent of $\psi \approx 2.6$.

the stretched exponential behavior predicted in Eq. (10) for the time dependence of ρ at the clean critical coupling $\lambda = \lambda_c^0 = 1$. We find that the data indeed follow a stretched exponential with an exponent of approximately 0.32, in excellent agreement with the analytical prediction $d_r/(d_r+z) = 1/3$. For $\lambda > \lambda_c^0$, the decay initially follows the stretched exponential, but eventually the density approaches its finite steady state value. The inset of Fig. 2 shows a log-log plot of $\rho(t) - \rho_{\text{steady}}$ vs t. The data clearly display power-law behavior in agreement with Eq. (11). A fit to this equation gives an exponent value of $\psi \approx 2.6$.

To summarize, we have demonstrated that extended impurities destroy the sharp DP phase transition in the contact process by smearing. In the remaining paragraphs we discuss the generality of our findings as well as their relation to the Griffiths effects [14,29]. The origins of Griffiths effects and the smearing found here are very similar; both are caused by rare large spatial regions that are locally in the ordered phase. The difference between them is a result of disorder correlations. For uncorrelated disorder, the rare regions are of finite size and cannot undergo a true phase transition. Instead, they fluctuate slowly. In contrast, if the rare regions are infinite in at least one dimension, a stronger effect occurs: each rare region can independently develop a nonzero steady state density. This leads to a smearing of the global transition.

The smearing mechanism found here relies only on the existence of a true phase transition on an isolated rare region. It should therefore apply not only to the DP universality class, but to an entire family of nonequilibrium universality classes for spreading processes and reaction-diffusion systems. Note that, while the presence or absence of smearing is universal in the sense of critical phenomena (it depends on symmetries and dimensionality only), the functional form of the density and other observables is *not* universal, it depends on the details of the disorder distribution [31]. Smearing phenomena similar to the one found here can also occur at equilibrium phase transitions. At quantum phase transitions in itinerant electron systems, even pointlike impurities can lead to smearing [32]. In contrast, for the classical Ising (Heisenberg) universality class, the impurities have to be at least 2D (3D) for the transition to be smeared, which makes the phenomenon less likely to be observed [33].

In conclusion, extended defects destroy the DP transition by smearing and lead to a (nonuniversal) exponential dependence of the density and other quantities on the control parameter. We suggest that this disorder-induced smearing may be related to the striking absence of DP scaling [11] in at least some of the experiments.

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